MODELLING THE COASE THEOREM

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More than fifty years ago Ronald Coase published 'The Problem of Social Cost'. In his paper, Professor Coase presents an intriguing idea that has since become known among economists and lawyers as the 'Coase Theorem'. Unlike most modern forms of economic analysis, however, Coase's Theorem is based on a verbal argument and is almost always proved arithmetically. That is to say, the Coase Theorem is not really a theorem in the formal or mathematical sense of the word. Our objective in this paper, then, is to remedy this deficiency by formalizing the logic of the Coase Theorem. In summary, we combine Coase's intuitive insights with the formal methods of game theory.

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I. INTRODUCTION

More than fifty years ago Ronald Coase published his seminal paper 'The Problem of Social Cost'.[1] In his paper, Professor Coase presents an intriguing idea that has since become known among economists and lawyers as the 'Coase Theorem'.[2] Unlike most modern forms of economic analysis, however, the Coase Theorem is based on a verbal argument and is almost always proved arithmetically. That is to say, Coase's Theorem is not really a *theorem* in the formal or mathematical sense of the word. Our objective in this paper is to remedy this deficiency by presenting the Coase Theorem as a formal game. In summary, we try to combine Coase's intuitive insights with the formal methods of game theory.

The remainder of this paper is organized as follows. Sections 2 and 3 provide some background regarding the Coase Theorem. Specifically, Section 2 briefly discusses the significance of the Coase Theorem, while Section 3 presents two of the most famous illustrations of the Coase Theorem—Coase's simple model of farmer-rancher interactions and Coase's arithmetical analysis of the problem of railway sparks—as well as some previous attempts to formally model the Coase Theorem. Next, Section 4 presents a general game-theoretic model of the Coase Theorem, one that does not depend on artificial parameter values. Specifically, Section 4.1 presents a simple two-player 'Coasian game' with probabilistic payoffs, Section 4.2 presents a population model of the Coase Theorem with probabilistic payoffs, and Section 4.3 then models an alternative Coasian farmer-rancher population game with high transaction costs and the presence of legal rules, but with fixed instead of probabilistic payoffs. Section 5 concludes and identifies some areas for future research.

II. BRIEF BACKGROUND: THEORETICAL SIGNIFICANCE OF THE COASE THEOREM

Before proceeding, it is worth taking a moment to explain the wider significance of the Coase Theorem in 'law and economics' and legal studies generally. From a theoretical or academic perspective, the Coase Theorem

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[1] RH Coase, 'The Problem of Social Cost' (1960) 3 JLE 1.
[2] ibid 2-8. See also RH Coase, 'The Federal Communications Commission' (1959) 2 JLE 1, 25-26.

is crucial to economic analysis of law. According to Richard Posner, for example, 'The most celebrated application of the concept of opportunity cost in the economic analysis of law is the Coase Theorem'.[3] Remove or disprove the Coase Theorem, and the economic approach to law is reduced to intellectual rubble or just another untestable or normative legal theory.[4] But with Coase's logical Theorem as its underlying theoretical foundation, the economic approach not only provides a clear and cogent lens for engaging in descriptive work and for understanding the effect of law on markets; at the same time, it also offers a powerful and forward-looking program for explaining and reforming almost all aspects of the legal system as well as myriad legal institutions, including property rights, tort law, and contracts.[5]

Moreover, the Coase Theorem has major theoretical and even practical implications as well by exposing the 'reciprocal' nature of economic externalities.[6] That is, the Coase Theorem substitutes the conventional 'victim-wrongdoer' paradigm prevalent in legal studies and moral philosophy with an entirely new and non-normative view of reciprocal conflict.[7] Consider a conflict situation between two parties, A and B. Instead of trying to identify the victim and the wrongdoer to the conflict—the traditional and still dominant method for analyzing conflicts and externalities in both the legal and economics literature—the Coasian approach invites one to see the conflict between A and B as a function of both parties' behavior. On this view, the Coase Theorem is nothing less

[3] Richard A Posner, *Economic Analysis of Law* (3rd edn, Little Brown 1986) 7.

[4] For a small but fairly representative sample of stinging criticisms of the Coase Theorem over the years, see Andrew Halpin, 'Coase's World and Coase's Blackboard' (2011) 31 EJLE 91; Dan Usher, 'The Coase Theorem Is Tautological, Incoherent, or Wrong' (1998) 61 Economic Letters 3; Paul Samuelson, 'Some Uneasiness with the Coase Theorem' (1995) 7 Japan World Economy 1; Daniel Q. Posin, 'The Coase Theorem: If Pigs Could Fly' (1990) 37 Wayne LR 89. For a thoughtful critique (in French) that the Coase Theorem is not a 'theorem', see Elodie Bertrand, 'Le théorème de Coase, une réflexion sur les fondements microéconomiques de l'intervention publique' (2002) 41 Perspectives de la Vie Économique 111.

[5] See generally Stephen G Medema, 'Legal Fiction: The Place of the Coase Theorem in Law and Economics' (1999) 15 Economics & Philosophy 209. See also FE Guerra-Pujol, 'Coase's Paradigm' (2010) 1 Indian JLE 1, 27-30.

[6] The word 'reciprocal' appears for the first time in the economics literature in Coase (n 1) 2 and in Coase (n 2) 26. See also Guido Calabresi, 'Neologisms Revisited' (2005) 65 Maryland LR 736, 738.

[7] For a novel application of Coase's reciprocal conflict idea to a science-fiction context, see FE Guerra-Pujol and Orlando I Martinez-Garcia, 'Clones and the Coase Theorem' (2011) 2 JL Social Deviance 43.

than a paradigm shift, a new way of looking at conflict situations.[8] Before Coase, the central question in legal studies used to be: Who is responsible for the harm? After Coase, the interesting and relevant question becomes: Who can mitigate or avoid the harm at the lowest cost to society? And thus one of the most intriguing and counterintuitive insights of the Coase Theorem is that, oftentimes, it is the ostensible victim who can avoid the harm at the lowest cost.

III. Coase's arithmetical models of the Coase Theorem (stray cattle and railway sparks)

Given the theoretical importance of the Coase Theorem, we present some simple game-theoretic models of Coase's Theorem in Section 4 of the paper. Since our models of the Coase Theorem are based in large part on Coase's analysis of the problem of railway sparks and his model of farmer-rancher interactions,[9] we briefly review the most salient features of Coase's models in subsections 3.1 and 3.2 below.

1. *Stray Cattle*

We begin by discussing Coase's farmer-rancher model, or what one scholar has dubbed 'the Parable of the Farmer and the Rancher'.[10] Coase introduced this model in his classic paper 'The Problem of Social Cost' to provide a vivid and concrete illustration of 'the problem of harmful effects'.[11] Although Coase's social cost paper contains many other examples of the problem of harmful effects—such as railway sparks, airplane noise, and smoking chimneys—it is the farmer-rancher problem that has captured the imagination of many scholars. Here, we describe the essential features of Coase's farmer-rancher model and summarize Coase's results in order to place our models of the Coase Theorem in their proper context.

Coase presents his farmer-rancher model in the opening pages of his social

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[8] For an extended discussion of 'paradigms' and 'paradigm shifts', see Thomas S Kuhn, *The Structure of Scientific Revolutions* (3rd edn, University of Chicago Press 1996) 77-91. See also Guerra-Pujol (n 5) 1-7.

[9] Coase (n 1) 2-8 (stray cattle), 29-34 (railway sparks).

[10] Robert C Ellickson, *Order without Law: How Neighbors Settle Disputes* (Harvard University Press 1991) 2. See also Robert C Ellickson, 'Of Coase and Cattle: Dispute Resolution Among Neighbors in Shasta County' (1986) 38 Stanford LR 623, 624-629.

[11] Coase (n 1) 1. Notice that the problem of harmful effects is more often referred to as 'negative externalities' or 'spillover effects' in the economics literature and is an important theoretical and practical problem in legal studies and in economics. For the standard economic analysis of harmful effects or 'negative externalities', see Paul A Samuleson and William D Nordhaus, *Economics* (19th edn, special India edn, McGraw-Hill 2010) 44-45.

cost paper as follows: 'Let us suppose that a farmer and a cattle-rancher are operating on neighboring properties. Let us further suppose that, without any fencing between the properties, an increase in the size of the cattle-rancher's herd increases the total damage to the farmer's crops'.[12] In other words, although the rancher's business is socially useful, his cattle-ranching activities may harm his neighboring farmer because stray cattle may often invade the farmer's land and destroy the farmer's crops. Coase also notes that this harm increases with the size of the rancher's herd, and he illustrates the link between the magnitude of the externality and the size of the rancher's herd with a simple arithmetical table.[13]

Next, having framed the essence of the problem—cattle versus crops—Coase isolates the two most essential features of his model: transaction costs, and institutions or legal rules. Generally speaking, transaction costs refer to the costs of negotiating and enforcing a fencing agreement between the farmer and rancher. Notice that transaction costs are either high or low relative to the costs of the externality to be avoided, that is, the value of the damaged crops when stray cattle invade the farmer's land. In general, transaction costs are low when the private costs of reaching and enforcing a fencing agreement are less than the costs generated by the externality. By contrast, transaction costs are high when the costs of the fencing agreement exceed the harm to be avoided.

Institutions refer to the rules of the game, that is, the rules of legal liability for crop damage caused by stray cattle. In this case, there are two possible institutions or legal rules to deal with the problem of stray cattle: a 'fence-in' rule, or an alternative 'fence out' rule. In summary, the fence-in rule is pro-farmer because it imposes liability for crop damage on the rancher. The rancher must fence-in his cattle or he will be liable for the crop damage caused by his stray cattle. Thus the rancher assumes the cost of fencing under the fence-in rule. The fence-out rule, by contrast, has the opposite effect. It is a pro-rancher rule because it imposes the cost of fencing on the farmer instead of the rancher: it is the farmer who is required to fence-out his neighbor's cattle under a fence-out regime.

In summary, Coase's farmer-rancher model is thus useful for two reasons. First, his model isolates two key variables—transaction costs and legal rules—and asks, what effect, if any, will these variable have on the allocation of resources among crops and cattle? Given these two key variables, there are four possible scenarios in all:

[12] Coase (n 1) 2-3.

[13] ibid 3.

Scenario #1	Low Transaction Costs and a Pro-Farmer Rule (fence-in)
Scenario #2	Low Transaction Costs and a Pro-Rancher Rule (fence-out)
Scenario #3	High Transactions Costs and a Pro-Farmer Rule (fence-in)
Scenario #4	High Transaction Costs and a Pro-Rancher Rule (fence-out)

Second, Coase's model is falsifiable, for Coase is, in effect, making a prediction or conjecture regarding what effect these two basic variables will have on the total allocation of resources (ie cattle versus crops). Moreover, the results of Coase's model are startling and surprising: the allocation of resources will depend entirely on the presence or absence of transaction costs and not on the legal rules, and it is this counterintuitive conclusion that is referred to formally as the 'Coase Theorem' in the academic literature.

Nevertheless, although the logic of Coase's model is unassailable, the premises of his model, such as the existence of transaction costs, are not stated formally or expressed mathematically. And although Coase relies on a simple arithmetical table to illustrate the logic of his model, the parameter values in his make-believe arithmetical table are arbitrary and artificial, a problem that plagues most restatements of the Coase Theorem.

2. *Railway Sparks*

Next, we turn to Coase's analysis of railway sparks, for Coase himself devotes considerable space in his social cost paper to the problem of railway sparks.[14] In summary, Coase introduces the problem of railway sparks by reference to 'Pigou's example of uncompensated damage to

[14] As an aside, it is interesting to note that Professor Coase devotes as much space in his social cost paper to railway sparks (about five full pages) as he does to the problem of stray cattle (seven pages). See Coase (n 1) 2-8 (cattle trespass), 29-34 (railway sparks). On a more personal note, the author also fondly recalls that his torts professor, Guido Calabresi, often referred to the problem of railway sparks in his lectures on tort law during the fall semester of the 1990-1991 academic year at Yale Law School.

surrounding woods by sparks from railway engines'.[15] [16] That is, the problem here is that (i) railway lines run through agricultural lands, and (ii) locomotive engines, especially when they run at high speeds, emit dangerous sparks, and (iii) these sparks may, in turn, produce destructive fires.

Coase's analysis of railway sparks—like his analysis of cattle trespass—is insightful, creative, and surprising. In place of a static analysis of the problem, Coase recognizes that the problem of railway sparks is really a strategic one, for the extent of the harm or damages caused by such sparks is the product of a joint interaction.[17] In summary, the harm caused by railway sparks is not only a function of economic decisions made by the railway company, such as whether to install spark preventers or the number of trains to run per day. This harm is also a function of decisions made by the landowners of property adjoining the railway line, such as whether to plant fire-resistant crops or whether to take their lands out of cultivation. Thus, although the problem of railway sparks appears different from the problem of cattle trespass, Coase correctly shows that, from an economic or social cost perspective, both problems are reciprocal and logically the same.

Despite the originality of his analysis, however, Coase does not really present a formal model of the problem of railway sparks, nor does he present a formal mathematical model of harmful effects or externalities generally. Instead, Coase illustrates his analysis of railway sparks with an arithmetical example. Coase himself, however, appears to recognize the limitations of his arithmetical analysis when he states, 'Of course, by altering the figures, it could be shown that there are other cases in which it would be desirable that the railway should be liable for the damage it causes'.[18]

3. *Some Non-Arithmetical Models of Coase's Theorem*

Lastly, before proceeding, we briefly review some previous attempts to

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[15] Coase (n 1) 30. As an aside, Pigou was an English economist who had written an influential treatise on welfare economics. See AC Pigou, *The Economics of Welfare* (4th edn, Macmillan 1932).

[16] It is worth noting that Coase refers to 'Pigou's example' not for its own sake, but rather to refute Pigou's approach to economics. In this paper, however, we will not enter into this fray, ie the details of Pigou's approach. For a summary of Pigou's approach, and a critique of Coase's critique of Pigou, see Herbert Hovenkamp, 'The Coase Theorem and Arthur Cecil Pigou' (2009) 51 Arizona LR 633. See also Calabresi (n 6) 738.

[17] Or, in Coase's own words, the problem is a 'reciprocal' one. Coase (n 1) 2.

[18] ibid 33-34.

model Coase's Theorem to set the stage for our models of the Coase Theorem in s 4 below. In summary, although some scholars have tried to formally model the Coase Theorem or test it experimentally, we explain why these previous approaches are deficient.

The literature on the Coase Theorem is vast[19]; in addition, this literature is highly polarized: for every paper in defense of the Coase Theorem, it seems, there is a paper critical of Coase's Theorem. But within this contentious Coasian corpus, formal or analytical models of the Coase Theorem are few and far between. Instead, most analyses, explanations, and extensions of the Coase Theorem (including both defenses and criticisms of Coase's Theorem) are expressed either in arithmetical terms or simply in words.[20]

One notable and early exception, however, is Posner, who presents a graphical analysis of the problem of railway sparks.[21] Since Posner's model is analytical, like the models we present in this paper, it is more general than most statements of the Coase Theorem, which rely on artificial parameter values or fanciful arithmetical tables. The problem with Posner's model, though, is that it is not really Coasian in spirit because his model assumes that only one of the parties is able to avoid the externality in his model. In summary, Posner models the problem of railway sparks in

[19] See, for example, Steven G Medema, 'The Coase Theorem' in Cary L Cooper and Chris Argyris (eds), *The Encyclopedia of Managerial Economics* (Basil Blackwell 1996). In addition, Professor Coase's social cost paper is (still) the most-cited law review article of all time. See Fred R Shapiro and Michelle Pearse, 'The Most Cited Law Review Articles of All Time' (2012) 110 Michigan LR 1483, Table I, 1489 and 1504.

[20] Most of the academic literature in this field restates the Coase Theorem arithmetically with arbitrary or make-believe values. For a small sample this literature, see Varouj Aivazian and Jeffrey L. Callen, 'The Coase Theorem and the Empty Core,' (1981) 24 JLE 175, 176-179; Kenneth R Vogel, 'The Coase Theorem and California Animal Trespass Law' (1987) 16 JLS 149, 159; Robert Cooter, 'The Cost of Coase' (1982) 11 JLS 1, 2-4; A Mitchell Polinsky, *An Introduction to Law and Economics* (2nd edn, Little Brown 1989) 11-14; Stewart J Schwab, 'Coase, Rents, and Opportunity Costs' (1991) 38 Wayne LR 55, 73-74; Stephen G Medema, 'Legal Fiction: The Place of the Coase Theorem in Law and Economics' (1999) 15 Economics & Philosophy 209, 214-215. Likewise, for a small sample of the literature in which the Coase Theorem is expressed exclusively in words, see, for example, Richard A Epstein, 'A Theory of Strict Liability' (1973) 2 JLS 151; George J Stigler, 'Two Notes on the Coase Theorem' (1989) 99 Yale LJ 631; Michael R Butler and Robert F Garnett, 'Teaching the Coase Theorem: Are We Getting It Right?' (2003) 31 Atlantic Economic J 133, 133-135; FE Guerra-Pujol and Orlando I Martinez-Garcia, 'Clones and the Coase Theorem' (2011) 2 JL Social Deviance 43, 65-81.

[21] Richard A Posner, *Economic Analysis of Law* (7th edn, Aspen 2007) 52-54. As an aside, Posner's elegant model first appears in print in the third edition of his textbook. See Posner (n 3) 44-46.

which sparks emitted by railroad locomotives cause fires that destroy crops, since the crops of some farmers are planted next to the railroad tracks, that is, within close range of the flying, fire-causing sparks. Posner states that 'changing the number of trains is assumed to be the only way of changing the amount of crop damage'.[22] But Posner's assumption misses the whole point of Coase's analysis, the idea that harms are 'reciprocal': a harm is the product of a joint interaction, such as the railroad company's decision to run a given number of trains per day and the farmer's decision not to plant fire-resistant crops.[23]

Aside from Posner, a few other scholars have also presented non-arithmetical models of the Coase Theorem. Among the most promising such models, we would point out the formal models of Lee and Sabourian,[24] Anderlini and Felli,[25] Acemoglu,[26] and Hurwicz.[27] Leonid Hurwicz, for example, presents an elegant formal of the Coase Theorem, but his model, however, is of limited scope and usefulness, since it assumes zero transaction costs, and as Coase himself has noted, most Coasian interactions (or 'Coasian games') will most often occur under conditions of high transaction costs.[28]

Some scholars have focused on the problem of transaction costs and have tried to formally model the process of Coasian bargaining. For instance, Lee and Sabourian model Coasian interactions as a dynamic bargaining game.[29] In summary, Lee and Sabourian demonstrate that such interactions produce a large number of equilibria and conclude that the Coase Theorem is valid if and only if there are no transaction costs. Of course, in real-world interactions, strategic considerations may often

[22] Posner, *Economic Analysis* (n 21) 53.
[23] That is why our models of the Coase Theorem (see s 4 below) assume, unlike Posner's model, that either party (not just the railroad company, for example) can take steps to avoid or reduce the risk of harm.
[24] Jihong Lee and Hamid Sabourian, 'The Coase Theorem, Complexity, and Transaction Costs' (2007) 135 J Economic Theory 214.
[25] Luca Anderlini and Leonardo Felli, 'Transaction Costs and the Robustness of the Coase Theorem' (2006) 116 Economic J 223.
[26] Daron Acemoglu, 'Why Not a Political Coase Theorem?' (2003) 31 J Comparative Economics 620.
[27] Leonid Hurwicz, 'What Is the Coase Theorem?' (1995) 7 Japan World Economy 49.
[28] See, for example, Ronald H Coase, 'The Coase Theorem and the Empty Core: A Comment' (1981) 24 JLE 183, 187.
[29] Lee and Sabourian (n 24). Their dynamic bargaining model of the Coase Theorem is based on the work of Rubinstein. See Ariel Rubinstein, 'Perfect Equilibrium in a Bargaining Model' (1982) 50 Econometrica 97. See also Lutz-Alexander Busch and Quan Wen, 'Perfect Equilibria in a Negotiation Model' (1995) 63 Econometrica 545.

obstruct Coasian bargains, even when transaction costs are low, especially in situations of bilateral monopoly.[30] For their part, Anderlini and Felli model 'Coasian negotiations' as a two-stage bargaining game with ex ante negotiation costs and show that such ex ante costs may produce a hold-up problem, thus preventing the parties to the negotiations from reaching an efficient Coasian bargain.[31] Their model, however, is also of limited usefulness, since in many real-world interactions, Coasian bargains are often made even under the conditions of their model.

Next, we wish to say a few words regarding Acemoglu's fascinating bargaining model in his 2003 paper, 'Why not a political Coase Theorem?'[32] Although some scholars have attempted to extend the domain of the Coase Theorem to certain specified areas of politics,[33] Acemoglu presents a generalized model of Coasian interactions between rulers and citizens. In essence, Acemoglu presents a model of political bargaining and shows that the applicability of the Coase Theorem to politics is limited because of commitment problems inherent to the political process.[34] But to the extent such commitment problems can be solved, the conclusions of the Coase Theorem would apply, even to the domain of politics.

In any case, it is worth noting that these various bargaining models of Coasian interactions are not really models of the Coase Theorem per se, for these approaches model the decision whether to negotiate and whether to make a Coasian bargain; that is, they model ex post behavior after the externality has occurred. Coase, in contrast, was not concerned with ex post bargaining per se; he was concerned with the ex ante problem of harmful effects, that is, with avoiding or reducing externalities ex ante, either through legal rules or through Coasian bargaining. That is why our models of the Coase Theorem (see s 4 below) are *ex ante* models, not *ex post* models. In other words, we model the decision whether to produce the externality in the first place.

Other scholars, in contrast, have taken an experimental or behavioral approach to the Coase Theorem.[35] That is, instead of attempting to model

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[30] See Robert Cooter, 'The Cost of Coase' (1982) 11 JLS 1.
[31] Anderlini and Felli (n 25).
[32] Acemoglu (n 26).
[33] See, for example, J Gregory Sidak, 'The Inverse Coase Theorem and Declarations of War' (1991) 41 Duke LJ 325.
[34] For an overview of the commitment problem, see chapter 2 of Thomas C Schelling, *The Strategy of Conflict* (rev edn, Harvard University Press 1980).
[35] For a small sample of this experimental literature, see Daniel Kahneman, Jack L Knetsch, and Richard H Thaler, 'Experimental Tests of the Endowment Effect and the Coase Theorem' (1990) 98 JPE 1325; Glenn W Harrison and Michael McKee,

the Coase Theorem formally, these researchers have tried to test the Coase Theorem experimentally. In summary, these experimental studies purport to test whether Coasian bargains will occur under artificial bargaining conditions with low transaction costs. The problems with the design and implementation of these experimental studies, however, are legion. Among other things, the main problems or design defects with these experimental tests of the Coase Theorem are that the objects subject to bargaining are low-value items, their prices are not set by markets but rather by the authorities conducting the experiments, and the human subjects participating in these experiments are not drawn from a random sample of the population.

Therefore, in place of artificial experimental studies, or complex *ex post* bargaining models, or verbal restatements of the Coase Theorem, or arithmetical analysis with arbitrary values, or instead of simply assuming that the Coase Theorem is true (as the late George Stigler would do[36]), in the remainder of this paper we present a simple analytic and game-theoretic treatment of Coasian games and the Coase Theorem.

IV. COASIAN GAMES

In this paper, a 'Coasian game' refers to any interactive, strategic, or game-theoretic model in which the interests of the players are conflicting due to the presence of negative externalities or harmful effects, such as stray cattle, airplane noise, or railway sparks. First, we present a simple two-player Coasian game in s 4.1 of the paper. Next, we present an even more generalized population model of Coasian interactions in s 4.2. Lastly, we return to Coase's simple model of farmer-rancher interactions and present an alternative farmer-rancher game in s 4.3 below.

1. *A Two-Player Coasian Game with Probabilistic Payoffs*

Our two-player Coasian game consists of a simultaneous-move game in which the players, whom we designate abstractly as Player A and Player B, share a simple strategy set: *cooperate* or *defect*. Our model is based on the following intuition: in the real world, when a person or a firm is engaged in a socially-useful activity, such as cattle ranching, his activity may produce a probabilistic risk of harm. For example, cattle may trespass on a neighboring farm and damage the farmer's crops, unless such crops are resistant to cattle, or a railroad locomotive may emit sparks and produce a

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'Experimental Evaluation of the Coase Theorem' (1985) 28 JLE 653; Elizabeth Hoffman and Matthew L Spitzer, 'The Coase Theorem: Some Experimental Tests' (1982) 25 JLE 73.

[36] George J Stigler, 'Two Notes on the Coase Theorem' (1989) 99 Yale LJ 631.

fire, unless landowners next to the railroad tracks avoid storing inflammable substances, such as hay, too close to the railroad tracks. Each player in our model must thus decide whether to *cooperate* by paying a cost to avoid or reduce the risk of a harm, such as damaged crops or to *defect* by not paying any harm-avoiding costs.

Before proceeding, we wish to make an important observation about our model: *both* players—not just the player who is 'causing' the harm in the traditional sense—are able to cooperate by taking steps to avoid harming the other player. For example, consider again Coase's problem of cattle trespass. The rancher can avoid harming the farmer by fencing-in his cattle, but the farmer himself can avoid this harm by fencing-out the cattle or by growing cattle-resistant crops. Likewise, with respect to the problem of railway sparks, the owner of the railroad company may reduce the risk of fires by reducing the speed of the locomotives or by installing costly spark-arresters, but at the same time, landowners can also reduce the risk of fire by not storing any inflammable substances next to the railroad tracks. The larger point is that (i) all these risk-reducing or harm-avoiding measures are costly cooperative measures and (ii) both players (not just the harm-producing player) must decide whether to cooperate or defect. If a player decides to cooperate, that means he is willing to pay a cost to avoid harming the other player; if, however, a player decides to defect, that means he is not willing to pay such a cost and is, in effect, creating a risk that the other player will be harmed.

Now, returning to our Coasian game, recall that both players in our model have to decide whether to cooperate (invest in a harm-avoidance measure to reduce the risk of an externality) or defect (make no such investment in risk reduction). Given this simple strategy set, and given that there are only two players, there are four possible scenarios or Coasian interactions in this Coasian game:

Scenario #1	both players cooperate: a 'cooperation-cooperation' interaction
Scenario #2	player A defects, while player B cooperates: a 'defection-cooperation' interaction
Scenario #3	player A cooperates, but player B defects: a 'cooperation-defection' interaction
Scenario #4	both players defect: a 'defection-defection' interaction

Since this is a game-theoretic model, the payoffs depend on the strategies simultaneously chosen by the players at the beginning of our Coasian game, and the payoffs associated with each possible interaction of the game may be expressed in 'normal form' as follows:[37]

	Player B *cooperate*	Player B *defect*
Player A *cooperate*	$b-c_1-pc_2$	$b-c_1-pc_2$
Player A *defect*	$b-pc_2$	$(1-p)(-c_2)$

Figure 1

Normal-form payoff table.

where c_1 is the cost of avoiding a given harm (ie the cost of investing in a safety device, such as a spark arrester, or the cost of reducing one's activity level, such as running fewer trains); where c_2 is the cost of the harm if such harm occurs (ie crop damage caused by fires); where p is the probability of such harm occurring; and where b is the benefit of avoiding the harm.[38]

Now that we have defined strategy set of the players (*cooperate* or *defect*) and assigned payoffs, we shall explain the assumptions in our model and explain the logic of each possible interaction of our Coasian game as follows:

First, consider scenario #1: mutual cooperation. For illustration, assume player A is a landowner whose land adjoins a railroad line and player B is a railroad whose locomotives produce sparks. If both players cooperate by

[37] For simplicity, the payoffs expressed in this table are player A's payoffs (i.e the row player's) because player B's payoffs are the same as player's A payoffs when both players cooperate or when both players defect and are the exact opposite of player's A payoffs when the players play different strategies.

[38] Also, notice that the payoffs of this Coasian game – that is, the benefits and costs corresponding with each strategy – are expressed in abstract terms, rather than in arithmetical terms, in order to illustrate the underlying logic and structure of seemingly unrelated problems, such as the problem of cattle trespass, railway sparks, and other harmful effects. In addition, another advantage of expressing these values as abstract values is flexibility and generality; that is, our abstract model permits us to derive results for any actual value that these parameters might take.

investing in safety or reducing their activity levels,[39] then each player's payoff for mutual cooperation is equal to $b - c_1 - pc_2$, where c_1 is the cost of avoiding the harm, and pc_2 the cost of the harm (if it occurs) discounted by the probability of such harm occurring, and b the benefit of avoiding a given harm.[40] Moreover, notice that one of the terms, pc_2, is probabilistic in nature. The probabilistic nature of this cost distinguishes our model from many other game-theoretic models in law and economics in which costs (and payoffs) are usually fixed. In our model, by contrast, the payoffs are probabilistic because investment in a given harm-avoidance measure (eg spark arresters, fences, etc) merely reduces the probability that a harm will occur (eg damaged crops or the payment of money damages) but such investment does not eliminate this risk altogether.

Next, consider scenario #2, a mixed (defection-cooperation) interaction. If player A defects and player B cooperates,[41] then player A's 'temptation payoff' is $b - pc_2$, while player B's 'sucker's payoff' is $b - c_1 - pc_2$. The logic of these payoffs is as follows: player B receives a 'temptation' payoff $b - pc_2$ because he gets the benefit of player A's costly investment in harm-avoidance without having to pay this cost himself, but player A's payoff is $b - c_1 - pc_2$ because he ends up paying the cost of avoiding the harm. (Again, notice that the last term, pc_2, of both players' payoffs is probabilistic for the same reasons stated in the paragraph above.[42]) Now, in contrast to the scenario above, consider the converse situation (scenario #3). That is, if player A cooperates instead of defecting, and player B defects instead of cooperating, then the payoffs of the players are reversed: player A now receives the payoff $b - c_1 - pc_2$, while player B receives the temptation payoff $b - pc_2$ because in this case it is player B who avoids having to pay c_1, the cost of avoiding the harm.

Before proceeding, the reader may ask: if only one player is willing to invest in a costly harm-avoidance measure (as in scenarios #2 and #3 above), why does the term p, the probability of avoiding the harm, remain

[39] For example, player A, the landowner, cooperates by planting fewer crops next to the railroad line, while player B, the railroad, cooperates by installing costly spark arresters on its locomotives or by operating fewer locomotives.

[40] For player A, the landowner, b might consist of the value of reducing the risk of harm to his crops. For player B, the railroad, b might be value of avoiding the risk of a lawsuit from the landowner.

[41] For example, player A, the landowner, might decide to defect by planting inflammable crops next to the railroad line (thus increasing the risk of harm to his crops from railway sparks). In contrast, player B, the railroad, might nevertheless decide to cooperate by installing spark arresters to reduce the risk of fires.

[42] That is, player B's costly investment in a given harm-avoidance measure merely reduces the risk that such harm will occur, it does not eliminate this risk.

the same as when both players invest in harm-avoidance measures separately (as in scenario #1)? That is, why is the probability of avoiding a given harm constant? In our model, we assume for the sake of simplicity that when one player invests in safety, any additional investment in safety by the other player does not further reduce the probability of harm.[43] That is, we assume that when both players invest in safety, their joint investment is redundant.

Lastly, consider scenario #4: mutual defection. What are the payoffs when both players defect, that is, when neither player A, the landowner, nor player B, the railroad, invests in any harm-avoidance measure or reduces their activity levels? In our model, both players forego the benefit b and avoid paying the harm-avoidance cost c_1 and instead receive a mutual defection payoff consisting of $(1-p)(-c_2)$.[44] In essence, the players 'take their chances' when neither invests in safety or reduces their activity levels. The logic of this mutual defection payoff is as follows: when neither player is willing to invest in a costly harm-avoidance measure, then this set of choices creates a probabilistic risk that a harm will occur, and moreover, we further assume for simplicity that this probabilistic risk is equal to $1-p$. That is, we assume that if p is the probability of harm when at least one of the players pays a cost to avoid that harm, then the probability of harm must be $1-p$ when no one invests in safety.[45]

Given this payoff structure, and given our simplifying assumptions, which of these four Coasian scenarios is most likely to occur? Put another way, what is the optimal strategy or best response from the point of view of each Coasian player? Is there a stable Coasian equilibrium?

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[43] In reality, such additional investment in harm-avoidance may reduce the risk of harm by some linear or marginally-declining amount, but we make the assumption of redundancy to keep our Coasian model as simple as possible.

[44] Notice that the mutual defection payoff is a function of c_2, not c_1. As one anonymous referee of this paper noted, making the mutual defection payoff a function of c_1 is problematic (and artificial) because c_1 refers to the cost of prevention, not the cost of the harm.

[45] This assumption, however, is open to debate. For instance, as one anonymous referee of this paper noted: if the probability of crop damage is only 0.1 when one of the parties builds a fence, then the probability of crop damage without a fence is not necessarily 0.9. (It could very well be higher or lower than 0.9 depending on the specifics of the situation.) Nevertheless, we make this simplifying assumption for ease of exposition and convenience, since our general assumption is that investment in safety tends to reduce the probability of harm, while the lack of such investment tend to increase this probability. Also, notice that if the magnitude of the harm to be avoided were less than the cost of avoiding it, then it would not make sense to invest in the harm-avoidance measure in the first place.

For his part, Coase famously asserts in his social cost paper that the players will negotiate and strike a Coasian bargain to solve the reciprocal harm/harm-avoidance problem, but only when transaction costs are zero.[46] This is the core of the Coase Theorem. But what happens when transaction costs are high, or when strategic behavior prevents the formation of Coasian bargains even when transaction costs are low?

If we take another glance at the game tree or at the payoff table of our Coasian game, the equilibrium path is not obvious. Since the payoffs are probabilistic and are expressed in variables, it is difficult to tell whether there are any dominant or dominated strategies or what the best responses of the players are. As a result, we will re-introduce the concept of probability, as well as the related idea of an 'expected payoff', in order to solve this game and find the existence of any possible equilibria.

Consider player A first.[47] Player A's expected payoff from playing a given strategy (*cooperate* or *defect*) depends on the probability P that player B might also play the same strategy as well as the probability $1-P$ that player B might choose a different strategy.[48]

Recall that player A has two choices in his strategy set. If player A cooperates, he will obtain the payoff $b - c_1 - pc_2$ with probability P (ie the probability that player B also cooperates), and he will also obtain the same payoff, $b - c_1 - pc_2$, with probability $1-P$ (ie the probability that player B defects). Player A's expected payoff of cooperating, which can be written as $E(C)$, is expressed formally as follows:

$$E(C) = (b - c_1 - pc_2)(P) + (b - c_1 - pc_2)(1-P)$$
$$E(C) = Pb - Pc_1 - Ppc_2 + b - c_1 - pc_2 - Pb + Pc_1 + Ppc_2$$
$$E(C) = b - c_1 - pc_2$$
(1.1)

In other words, when player A cooperates by investing in safety or reducing his activity level, his payoff is constant regardless of what player B does. By contrast, player A's expected payoff from defecting does depend on what player A does. In summary, player A receives the payoff $b - pc_2$

[46] That is, when 'the pricing system works smoothly'. Coase (n 1) 5.
[47] In fact, the analysis in the remainder of this section applies equally to both players since, for as we stated earlier in n 37, the payoffs in our simple model are symmetrical.
[48] For reference, notice that this type of probability (ie the probability P of the other player's strategy selection) is written as a capital letter to distinguish it from the earlier type of probability, that is, the probability p that a harm will occur if one or both of the players invests in safety.

when player B cooperates and the payoff $(1-p)(-c_2)$ when Player B defects. Since player B will cooperate with probability P and defect with probability $1-P$, we can express player A's expected defection payoff $E(D)$ as follows:

$$E(D) = (b-pc_2)(P) + [(1-p)(-c_2)](1-P)$$
$$E(D) = Pb - Ppc_2 + (pc_2 - c_2)(1-P)$$
$$E(D) = Pb - Ppc_2 + pc_2 - c_2 - Ppc_2 + Pc_2$$
$$E(D) = Pb - c_2(2Pp + P + p - 1) \tag{1.2}$$

What if we assume that $pc_2 = 0$ for simplicity; that is, what if we assume that the probability of harm is low, close to zero, when at least one of the players invests in safety or reduces his activity level. Under this assumption, player A's expected cooperation payoff is

$$E(C) = b - c_1 \tag{1.1a}$$

and player A's defection payoff $E(D)$ becomes:

$$E(D) = Pb - c_2 + Pc_2$$
$$E(D) = P(b + c_2) - c_2 \tag{1.2a}$$

Notice that the size of player A's defection payoff (equation 1.2a) is a function mostly of P, the probability the player A will cooperate. By contrast, player A's cooperation payoff is a function only of the terms b and c_1. In other words, player A's best response depends mostly on what strategy player B chooses. If player B decides to cooperate, ie $P = 1$, then $E(D)$ will be greater than $E(C)$ because player A's expected payoff for defecting will be b, while his expected payoff for cooperating will remain $b - c_1$.

Now, assume that player B decides to defect, ie $P = 0$. In this case, $E(C)$ will be greater than $E(D)$ because player A's expected payoff for defecting will be $-c_2$, while his expected payoff for cooperating will remain $b - c_1$. Assuming that $b>c_1$,[49] Player A should cooperate when B defects, and conversely, player A should defect when B cooperates.

[49] This is a reasonable assumption, since otherwise, it would not make sense to invest in safety when the cost of such investment is greater than the benefit to be received from such investment.

In other words, our model shows that player A's decision to cooperate or defect is not so much a function of legal rules or transaction costs but of player B's choice, which in turn is a function of player A's decision.[50] Is there any way around this circular result?

One possible solution is to deny the Coase Theorem: in the absence of an equilibrium solution to our Coasian game, the choices of both players might then be a function of the legal rules, contra the invariance thesis of the Coase Theorem. On this view, legal rules are a device for coordinating the choices of the players, specifically, a device for getting at least one of the parties to invest in safety or reduce his activity level. This analysis also confirms a central axiom of law and economics, that the applicable legal rule should impose liability on the party with the lowest cost of avoiding the harm.[51] Thus, under the assumptions of our model, we would expect the rule of legal liability to matter, even under conditions of low transaction costs.

2. *An n-Player Coasian Game with Probabilistic Payoffs*

Next, we present a multi-player evolutionary model of our Coasian game. In summary, our *n*-player evolutionary game works as follows:

(a) There is a large and well-mixed population of players.

(b) This population contains two types of players, cooperators and defectors.

(c) At the start of each round of play, two players are selected at random from the population and then, during each round of play, these two players play a Coasian micro-game.

(d) After each round of play, the player with the highest payoff in the micro-interaction not only survives but also produces a descendant-clone who asexually inherits the victor's player's type (ie if the victor was a cooperator, then his descendant is a cooperator).

(e) The player with the lowest payoff, in contrast, is eliminated from the population.

[50] Also, notice that this analysis is independent of the level of transaction costs.
[51] For the classic 'cheaper cost avoider' theory of tort liability, see Guido Calabresi, *The Costs of Accidents: A Legal and Economic Analysis* (Yale University Press 1970).

(f) Lastly, if the interaction ends in a draw or tie (ie cooperator-cooperator or defection-defection interactions), both contestants survive but neither produces a descendant.

The purpose of this game is to determine which strategy will spread through our population of Coasian players. Will cooperators outperform defectors, or will defectors displace cooperators, or will the population consist of a stable mix of cooperators and defectors? To answer these questions, we proceed in several stages.

First, we restate the expected payoffs corresponding to each possible Coasian micro-game. In summary, there are four possible micro-interactions in the *n*-player evolutionary game, as in the traditional two-player model: (1) mutual cooperation, or C|C for short; (2) cooperation-defection, or C|D; (3) defection-cooperation, or D|C; and (4) mutual defection, or D|D. Since the structure of the payoffs in the *n*-player game are the same as in the two-player game, the payoffs corresponding to each Coasian micro-game are as follows:

E(C|C) = [the payoff to a cooperator given that he interacts with another cooperator] = $b - c_1 - pc_2$
E(C|D) = [the payoff to a cooperator given that he interacts with a defector] = $b - c_1 - pc_2$
E(D|C) = [the payoff to a defector given that he interacts with a cooperator] = $b - pc_2$
E(D|D) = [the payoff to a defector given that he interacts with another defector] = $(1 - p)(-c_2)$

In summary, on the far left- and far-right hand sides of the table above, we have expressed the payoffs corresponding to each Coasian micro-interaction in mathematical form, while in the middle section, separated by brackets [...], we have 'translated' the mathematical notation into plain English for the non-mathematical reader.

Moreover, since this is a population model, the success of a given strategy is said to be 'frequency dependent' because the success or survival rate of a strategy depends not only on the frequency of the other strategy but also on that strategy's own frequency.[52] Since success or 'fitness' (rate of survival) is frequency dependent, we proceed to use the methods of evolutionary game theory to determine whether a strategy is an

[52] For an illustration of frequency dependency, see Richard McElreath and Robert Boyd, *Mathematical Models of Social Evolution* (University of Chicago Press 2007) 38.

'evolutionarily stable strategy' or ESS and to find the long-run evolutionary equilibrium of the population—that is, the frequency of cooperators and defectors over many generations.[53] Specifically, we wish to answer the following key questions: (i) is cooperation an evolutionarily stable strategy or ESS? In other words, are cooperators able to resist invasion by defectors? (ii) Likewise, is defection an ESS? That is, are defectors able to resist invasion by cooperators? (iii) Or, do Coasian interactions produce an evolutionarily stable mix of cooperators and defectors?

Let P be the frequency of cooperators in the population, and thus let $1 - P$ the frequency of defectors in the population. First, consider a population in which the frequency of cooperators is very high $(P \approx 1)$. With this population structure, cooperators rarely interact with defectors because the frequency of defectors is low $(1 - P \approx 0)$, and thus the average fitness of a cooperator, written as $W(C)$, is determined by his interactions with other cooperators in the population as follows:[54]

$$W(C) = w' + 1[E(C|C)] + (1 - 1)[E(C|D)]$$
$$W(C) = w' + E(C|C) + 0$$
$$W(C) = w' + b - c_1 - pc_2$$

At this point, consider the appearance of a rare defector mutant in this population of cooperators. Will this defector be able to spread across the population, gradually displacing the cooperators, or will the cooperators be able to resist invasion by the defectors? To answer this question, we must determine the average fitness of the rare defectors among the population of cooperators, and then compare the average fitness of such defectors with the average fitness of cooperators. Since defectors are rare $(1 - P \approx 0)$, the chance one defector will meet another defector is likewise small. As a result, the average fitness of a defector, written as $W(D)$, is determined by his interactions with cooperators as follows:

$$W(D) = w' + 1[E(D|C)] + (1 - 1)[E(D|D)]$$
$$W(D) = w' + E(D|C) + 0$$
$$W(D) = w' + b - pc_2$$

Thus, when we compare the average fitness levels of the majority

[53] For an overview of the ESS concept, see generally John Maynard Smith, *Evolutionary Game Theory* (Cambridge University Press 1982). See also George R Price and John Maynard Smith, 'The Logic of Animal Conflict' (1973) 246 Nature 15.

[54] Before proceeding, note that the parameter w' in our equations refers to the 'baseline fitness' or baseline survival rate of all the individuals in the population—that is, the probability of survival from generation to generation—and thus reflects the strength of selection on a given population. See McElreath and Boyd (n 51) 40-41.

cooperators and the rare defectors, we see that defectors have a higher average fitness than cooperators. Stated formally, we see that $W(D) > W(C)$ because $b - pc_2 > b - c_1 - pc_2$.[55] This means that defectors will outperform cooperators and thus spread across and invade the population of cooperators.

But now this state of affairs raises a new question: can a population of defectors resist invasion by cooperators? Consider a population in which the frequency of defectors is high $(1 - P \approx 1)$. With this population structure, defectors interact with other defectors most of the time, so the average fitness of a defector, W(D), is determined by his interactions with other defectors as follows:

$$W(D) = w' + 1[E(D|D)] + (1 - 1)[E(D|C)]$$
$$W(D) = w' + E(D|D) + 0$$
$$W(D) = w' + (1 - p)(-c_2)$$
$$W(D) = w' + pc_2 - c_2$$
$$W(D) = w' + c_2(p - 1)$$

Next, consider the appearance of a rare cooperator mutant in this Hobbesian population of defectors. Will the rare cooperators be able to invade the population and displace the defectors, or will the defectors be able to resist invasion by the cooperators? To answer this question, we must compare the average fitness level of the rare cooperators with that of the majority defectors. Since cooperators are rare $(P \approx 0)$, the average fitness of a cooperator is thus determined by his interactions with defectors as follows:

$$W(C) = w' + 1[E(C|D)] + (1 - 1)[E(C|C)]$$
$$W(C) = w' + E(C|D) + 0$$
$$W(C) = w' + b - c_1 - pc_2$$

Now, when we compare the average fitness levels of the majority defectors and rare cooperators, we see that the rare cooperators have a higher average fitness than the majority defectors do. This result also raises an intriguing question: will the population of cooperators and defectors continue to cycle depending on which group is in the majority, or is there an evolutionarily stable mix of cooperators and defectors?[56]

In any case, how does this result relate to Coase's Theorem? In summary,

[55] Notice that the baseline fitness terms, w', cancel out.

[56] One could easily find for this equilibrium mix of defectors and cooperators by setting $W(C)$ equal to $W(D)$, substituting p' for p, and solving for p'.

our result shows another dimension of the Coase Theorem. Recall that Coase himself was concerned with negative externalities, or 'the problem of harmful effects'.[57] Most of the literature on the Coase Theorem focuses on transaction costs, legal rights, bargaining, the endowment effect, and willingness to pay, and thus most commentators tend to focus exclusively on law, behavioral economics, or on economics proper: the benefits and costs of various negative externalities, such as the harmful effects produced by cattle ranching, crop farming, railroads, and so forth. In brief, the Coase Theorem asks two basic questions: what is the harm, and who should pay the cost to avoid this harm? Thus, under traditional economic or Coasian analysis, once the harm has been identified, the main questions are always economic in nature: 'who pays whom?'

Our analysis, in contrast, raises a different set of questions. Instead of 'who pays whom?', our analysis asks: which harm-avoidance measure more effectively reduces the probability or risk of harm? Unlike traditional economic or Coasian analysis, our analysis shows that what really matters is not the (social or private) benefits generated by a conflicting activities and not the (social or private) costs imposed by such activities, but rather what really matters is the effectiveness of the harm-avoidance measures that are available to the parties to address a given harm, and this insight is captured by the probabilistic payoffs, namely, the parameter p, in our models of Coasian games.

This insight is not necessarily inconsistent with main results of the Coase Theorem: the invariance thesis and the efficiency thesis. For example, the efficiency criterion is consistent with the proposition that courts and legislatures should impose legal liability on the party that can most effectively reduce the probability of a given harm, but notice that our emphasis is not on the cost of avoiding a given harm but rather on the probability of avoiding such harm. In many cases, cost and probability will be close proxies for each other, but in other cases, these issues may diverge: the 'cheaper cost avoider' may not necessarily be able to reduce the probability of a given harm more effectively than another party might. In other words, a different party might able to reduce the risk of such harm more effectively (although at greater cost) than the designated cheaper cost avoider. This possibility opens up a new avenue of research, a new door for the Coase Theorem to open.

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[57] Coase (n 1) 1.

3. *An Alternative Coasian Game with High Transaction Costs, Fixed Legal Rules, and Deterministic Payoffs*

Lastly, we model a farmer-rancher game with high transaction costs (ie no Coasian bargaining among the players) and with fixed legal liability rules but with deterministic (non-probabilistic) payoffs. For this revised Coasian game, we now add the following assumptions:

(a) The population is large, well-mixed, and composed of two ideal types: farmers F and ranchers R.

(b) Individuals from this large, well-mixed population are selected at random and interact in pairs.

(c) Individuals are not permitted to make side deals or Coasian bargains with each other (that is, we assume high transaction costs).

(d) In the absence of a legal rule, no fence gets built.

(e) When a legal rule is enacted (either fence-in or fence-out), there is full compliance with the rule; that is, if the rule is fence-in, all cattle ranchers comply with the rule and fence-in their lands, and by the same token, if the rule is fence-out, all farmers comply with the rule and fence-out their lands.

(f) The cost of fencing is constant and the payoffs to farming and ranching are equal, or stated formally, $bR=bF$.

As before, we recognize that these simplifying assumptions are not necessarily consistent with real-world conditions. For example, in a real-world situation, the cost of fencing will vary depending on the size of one's land, and the revenues generated by farming and ranching will likewise vary depending on a wide variety of factors. Nevertheless, we make these artificial and unrealistic assumptions to simplify our mathematical analysis and test the main insight generated by the Coase Theorem: the conjecture that the rules of the game will have no effect on the allocation of resources when transaction costs are high.

Now, consider a large, well-mixed population consisting of farmers and ranchers. Ranchers receive a fixed payoff of bR, while farmers receive a fixed payoff of $bR - dp$, where d is the cost of the damages or harm to crops caused by stray cattle, p is the probability that this harm will occur (in the absence of a fence), and $dp > 0$. For now, assume there is no fencing rule or convention in place and that $bR=bF$.

Given this set of assumptions, we see that ranching is an evolutionarily stable strategy or ESS since the ranching payoff exceeds the farming payoff, since by definition $bR > bF - dp$. As a result, the population dynamic will be pro-rancher: when ranchers are common, farmers will not be able to invade a population of ranchers, and when farmers are common, ranchers will be able to invade the population and displace the farmers, and so either way, ranchers will always dominate the population in the absence of any fencing rule or convention.

But now consider what effect a fencing rule would have on our model. There are two possible rules: fence-in and fence-out. Assume both fencing rules are equally effective in solving the problem of stray cattle, so the main effect of either rule is simply to rearrange the payoff structure of farmer-rancher interactions, since fences are costly to build and maintain. Specifically, under a fence-in regime, the payoff to a rancher is $V(R)=bR - c$, where c is the cost of fencing-in the rancher's land, and likewise, the payoff to a farmer is $V(F)=bF - (1 - p)d$, where this last term is the probability that the farmer's crops are damaged even with a fence in place. To keep this model as simple as possible, we will assume that the fence-in rule neutralizes the problem of stray cattle, that is, we assume that $(1 - p)d = 0$. To recap, then, when ranchers are required to fence in their cattle, an individual rancher's payoff is reduced by the cost of fencing-in his land, while farmers receive a fixed payoff bF since the fence-in rule neutralizes the problem of stray cattle, ie $(1 - p)d = 0$.

Given a fence-in rule, we now see that farming will be an ESS because the farming payoff exceeds the ranching payoff, or $bF > bR - c$. Thus, under a fence-in regime, the proportion of farmers in the population will increase in frequency over time. This means the following population dynamic will occur: when farmers are common, ranchers will not be able to invade a population of farmers, but when ranchers are common, farmers will always invade the population and displace the ranchers.

Next consider, what happens when the applicable rule is fence-out, instead of fence-in. Under a fence-out regime, the payoff to a rancher is $V(R)=bR$, while the payoff to a farmer is $V(F)=bF - c$, since now it is the farmer who must pay the fencing costs.[58] In summary, given a fence-out rule, ranching will be an ESS because the ranching payoff exceeds the farming payoff, that is, $bR > bF - c$. As in the case with no legal rule, the population

[58] Again, for simplicity, we assume that $bR = bF$, ie, there is no reason to prefer farming over ranching or vice-versa, and we assume that the fence-out rule solves the stray cattle problem, so we ignore d.

dynamic will be pro-rancher: when ranchers are common, farmers will not be able to invade a population of ranchers, and when farmers are common, ranchers will always be able to invade the population and displace the farmers.

In summary, the lesson of this Coasian game is clear: the dynamic of the population over time is a function of the rules. That is, when transaction costs are high, or when Coasian bargaining is not possible, the payoffs of the players, and thus the outcome of the game, is dependent on the legal rule. This result thus confirms one of the conclusions or predictions of the Coase Theorem: when transaction costs are high, the choice of legal rule will determine the allocation of resources.

V. CONCLUSION

In closing, we concede that the Coasian games presented in this paper abstract from reality. Specifically, our models of the Coase Theorem are much more abstract and idealized than actual or real-life farmer-rancher interactions in many respects: the population of farmers and ranchers in our models are well-mixed and large, their corresponding strategies are simple and stylized, and the payoffs to each strategy are kept constant. In addition, we have omitted stochastic effects such as noise or errors from our model. Instead, we have decided to trade off realism for tractability. That is, we have intentionally designed our model of farmer-rancher interactions to be as simple as possible to illustrate the logic of the Coase Theorem.

We now wish to close this paper by looking towards the future and sketching some other possible Coasian games. Specifically, we briefly consider some variations to our model of the Coase Theorem and identify some new questions for future research:

Question #1 What happens when $bR \neq bF$?

One direction for future work is to relax the assumption of equal payoffs, such as making the payoffs vary inversely with the choice of legal rule. For example, with a pro-farmer fence-in rule, a rancher might respond by investing less in ranching (eg by decreasing his herd from n steer to $n - 1$ steer), while farmers might respond by investing more in farming (by planting more crops), and this change in investment levels will, in turn, affect the expected payoffs corresponding with each activity.

Question #2 What happens when the choice of legal rule is endogenous to the model?

That is, what happens when the players must not only decide how much to invest in farming or ranching but must also decide how much to invest in rent-seeking activities, such as lobbying or litigation, in order to obtain a favorable legal rule. Now, the payoffs of the players will be a function of their farming or ranching activities; their payoffs will also be a function of their lobbying or litigation activities as well, and since activities like lobbying and litigation tend to increase the probability of a favorable ruling, such a possibility also lends itself to a probabilistic analysis.

Question #3 What happens if we assume a different population structure?

That is, instead of assuming a large and well-mixed population, as we have done in this paper, what if we were to model the population structure graphically? For example, imagine a large number of evenly-sized towns distributed over a large square grid. Each town contains n number of plots of land with n number of farmers and ranchers, and in addition, each town must decide whether to adopt a pro-farmer rule (fence-in) or a pro-rancher rule (fence-out), with the choice of legal rule depending on which group is a majority in each town. Given this graphical configuration of the problem, we would then find what mix of pro-farmer and pro-rancher rules will result over the long run. That is, instead of modelling a population of farmers and ranchers, we would model a population of legal rules, with feedback effects between the population of legal rules and the population of farmers and ranchers in each town, since the choice of legal rule depends on the population dynamic in each town, and since the population dynamic in turn, depends on the choice of legal rule. Such an approach to the Coase Theorem, one with feedback loops between the legal rules and the economic activities of the actors, seems to be an especially promising area for future Coasian analyses.